9.3.2 Test of normality.

The discussed test can be applied only if in the original populations the considered values have a normal distribution. It is necessary to test this condition before the application of these tests. For the values $x_1, x_2, x_3, \dots x_n$ of a random sample the following tests indicate if they are from a population of a given distribution *f(x)*.

a) χ^2 test. A frequency table of the values is made. Let $f_1, f_1, \dots, f_k$ be the frequencies of the cells centered at $v_1, v_2, \dots v_k$. The value

$u = \sum_{i=1}^{k} \frac{(f_i - g(v_i))^2}{g(v_i)}$ where $g(v_i)$ is the value of the assumed theoretical probability function in the value v_i. In our case this is the normal probability function with a mean and deviation corresponding to the data.

If the data has normal distribution these values *u* have a χ^2 distribution with *k*-1 degrees of freedom.

Example 9.3.2.1

b) KS test. The χ^2 test requires the representation of the theoretical distribution in a discrete form. In the case of the normal distribution is more natural to use the Kolmogorov-Smirnov test. In the KS test the distribution (accumulated) function is used instead of the probability function. If $O(x)$ is the distribution of the observed data and $F(x)$ is the theoretical distribution (with the same distribution and deviation than the observed data) and the observed data are $x_1, x_2, x_3, \dots x_n$ then the following maximum is estimated:

$D = MAX(|F(x_i) - O(x_i)|)$

If this value is greater than the values in a table for the KS test the normality hypothesis is rejected.

Example 9.3.2.2

9.4. Sensitivity Analysis.
Sometimes is important to know how some specific results change with certain change in the input data. The reasons are:
1) It is important for practical or theoretical reasons to evaluate the influence that certain data has on the results.
2) The values of some parameters are not exactly known, and it is required to know how these uncertainties alter the results. This may lead to dismiss the effect or the indetermination, if the effects are negligible, or, if they are not, to further research for more exact values. In this sense sensitivity analysis is a guide to research.
The sensitivity analysis is usually made by changing one input variable at a time and determining the output by the methods described in 9.2. For the comparisons of the original values with the results of the changed models, the techniques used in 9.3 may be used.
In many cases the change in an output variable when an input variable is changed depends of the values of other variables. In this case is said that these influencing variable interact with the relation.

Example: As a trivial example consider the influence of the base b on the surface S of a rectangle of high a. As the surface is $S = b \times a$, the change of the surface for a change Δb of the base is $(b + \Delta b) \times a - ba = a\Delta b.$ The change depends of the value of a. The high interacts with the influence of the base on the surface.

When the variable z interacts with the influence of x on y, then determining each change of y for different values of x is not enough. These determinations must be done for different values of z. In 9.5 the methods to do systematically this analysis will be discussed.

The usual way to do the sensitivity analysis is by means of factorial experiments (see 9.5)
Use of interval arithmetic. Interval arithmetic (see Moore) consist in making the arithmetic operations with the lower and upper values of an indeterminate quantity in such a way that the maximum interval for the results are obtained..
If (a_1, a_2) and (b_1, b_2) are respectively the lower and upper values of a and b then:

$$(a_1, a_2) + (b_1, b_2) = (\min(a_1 + b_1, a_1 + b_2, a_2 + b_1, a_2 + b_2),\ (\max(a_1 + b_1, a_1 + b_2, a_2 + b_1, a_2 + b_2))$$
$$= (a_1 + b_1, a_2 + b_2)$$
$$(a_1, a_2) - (b_1, b_2) = (\min(a_1 - b_1, a_1 - b_2, a_2 - b_1, a_2 - b_2),\ (\max(a_1 - b_1, a_1 - b_2, a_2 - b_1, a_2 - b_2))$$
$$= (a_1 - b_2, a_2 - b_1)$$
$$(a_1, a_2) \times (b_1, b_2) = (\min(a_1 \times b_1, a_1 \times b_2, a_2 \times b_1, a_2 \times b_2),\quad (\max(a_1 \times b_1, a_1 \times b_2, a_2 \times b_1, a_2 \times b_2))$$
$$(a_1, a_2) / (b_1, b_2) = (\min(a_1 / b_1, a_1 / b_2, a_2 / b_1, a_2 / b_2),\quad (\max(a_1 / b_1, a_1 / b_2, a_2 / b_1, a_2 / b_2))$$

Multiplication and division cannot be simplified due to the sign rule.

Example.
(1,4)+(3,7)=(4,11)
(-5,-4)-(-8,-2)=(-3,4)
(-5,3)×(-3,4)=(-20,1a simultaneous change in C1 and C4the results are almost equal than those for C4 only.